Chapter 1

Strain gages and ideal bridges

TEC/NOT/001

This paper introduces the basics of strain gage theory and the terminology used. It examines how these gages can be used in a variety of ways with Wheatstone bridges, focusing on how strain can cause a change in resistance, which in turn causes a voltage change across the bridge that can be measured. Also, the relationship between the output voltage and the change in resistance is described for the more popular bridge configurations.

The following topics are discussed:

1.1 Stress and strain

The following figure displays a piece of metal fixed at one end and attached to a dangling mass at the other.

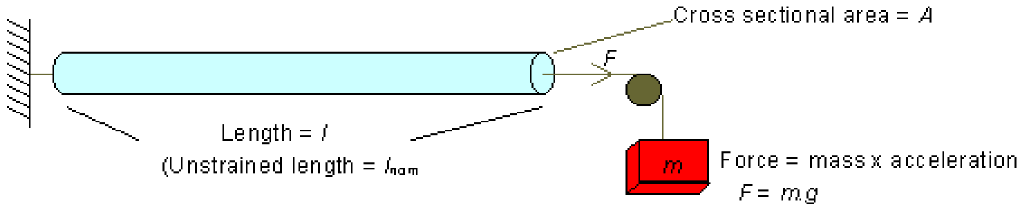


Figure 1-1: A beam under stress

The mass (m) causes a force (F), which places the beam under stress (σ), causing the beam to increase in length ($l_{nom} \rightarrow l$).

$$Stress \Rightarrow \sigma \equiv \frac{F}{A}$$

In this paper, "≡" denotes "equal by definition" and the subscript "nom" denotes "the unstrained/unstressed or nominal condition".

Strain is a measurement of the fractional change in length and is defined as:

$$Strain \Rightarrow \varepsilon \equiv \frac{l - l_{nom}}{l_{nom}}$$

For certain materials, there exists a small elastic range where the strain is linear with respect to stress. In particular Hooke's law states:

$$\varepsilon \cong \frac{\sigma}{E}$$

where *E* is Young's modulus.

If, as in the previous figure, the force is perpendicular to the cross-section and positive, then the force is said to be uniaxial and tensile. If the force is negative, then it is said to be compressive. If the force acts along the cross section, then it is said to be shear stress (see the following figure).

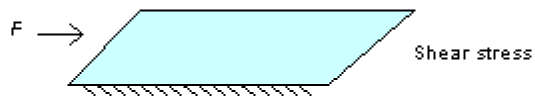


Figure 1-2: Shear stress

Typical strains are in the order of parts per million (ppm) and it is common to use the term $\mu\varepsilon$ (micro-strain) defined as follows:

$$\mu\varepsilon \equiv \varepsilon \cdot 10^6 = \frac{l - l_{nom}}{l_{nom}} \cdot 10^6$$

1.2 Strain and fractional change in resistance

The following figure exaggerates how the shape of a piece of metal changes with strain.

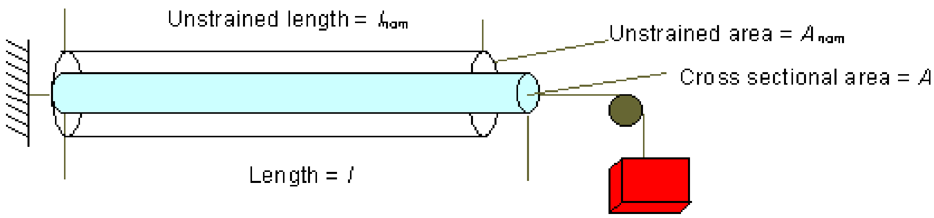


Figure 1-3: How the shape of a cylinder changes when stretched

The resistance of a piece of metal is proportional to its length (l) and inversely proportional to its cross sectional area (A). The volume (V), which remains constant, is a product of length and area (V=l.A). In other words:

$$R \propto \frac{l}{A}$$

The constant of proportionality is ρ, the material resistivity:

$$R = \frac{\rho l}{A} = \rho\frac{l}{V/l} = \rho\frac{l^2}{V}$$

Because strain is defined as a fractional change in length, the resistance (which is a function of length as just shown) can be used as a means of measuring strain.

The fractional change in resistance (δ) can be defined as:

$$\delta \equiv \frac{R - R_{nom}}{R_{nom}}(\Omega/\Omega)$$

Substituting the equation for R above (and canceling V and the resistivity constant) this becomes:

$$\delta = \frac{l^2 - l_{nom}^2}{l_{nom}^2} = \left(\frac{l}{l_{nom}}\right)^2 - 1$$

From the definition of strain ε:

$$\varepsilon \equiv \frac{l - l_{nom}}{l_{nom}} = \frac{l}{l_{nom}} - 1 \Rightarrow \frac{l}{l_{nom}} = \varepsilon + 1$$

Combining the last two equations gives a formula for fractional change as a function of strain:

$$\delta = (\varepsilon + 1)^2 - 1 = 2 \cdot \varepsilon + \varepsilon^2$$

For very small strains the second order term is sometimes ignored:

$$\delta \cong 2 \cdot \varepsilon$$

Hence the nominal gage factor (F_G) of 2 used for strain gages.

Even if the gage factor is not 2 (or is not even linear) it must be known (from manufacturers data sheets) or deduced (from experience or calibration). The rest of this paper, apart from a short discussion of errors, assumes the gage factor is known and concentrates on measuring δ.

1.3 Sources of error in the gage

The following sources of error are evident in strain gage systems:

- The bridge itself
- The measurement equipment

This section looks briefly at some of the errors that can exist in the bridge itself due to bonding and temperature. Other application notes look at errors in the measurement equipment.

1.3.1 Temperature errors

Strain gages on airplanes are rarely kept at a constant temperature. To measure the resistance, current is applied, which causes power dissipation (heating) in the gage.

Sometimes referred to as pseudo-strains, heating causes the following types of errors:

- As the gage, bonding and member change temperature, they expand or contract at different rates. In other words, with a constant stress the strain changes.
- The resistivity of the gage (and hence its resistance) changes with temperature.

These errors can be somewhat compensated for with a known gage, current, type of bonding and material by measuring the temperature. However this is not always practical.

Certain gages when used with specific current, bonding and material are designed to self compensate.

In bridge circuits advantage is often taken of the fact that the absolute resistance values of bridge arms is less important than the ratio (see “1.4 The Wheatstone bridge and Poisson’s ratio” on page 4) so that gages can have compensation arms (bonded perhaps at right angles to the strain being measured).

1.3.2 Bonding errors

Great care must be taken when bonding gages to a structure.

If the gage is not parallel to the strain being measured, this causes an error. For example, even being out by 2.5° causes approximately 0.1% gain error.

If the gage is not flat, it appears shorter with respect to the direction of strain, thus causing a gain error.

If the bonding material is not of the correct type and thickness, the heat dissipation will not be as expected and hence a strain error will be induced on the gage.

1.4 The Wheatstone bridge and Poisson's ratio

The bridge has two sides (left and right) and four arms (see the following figure).

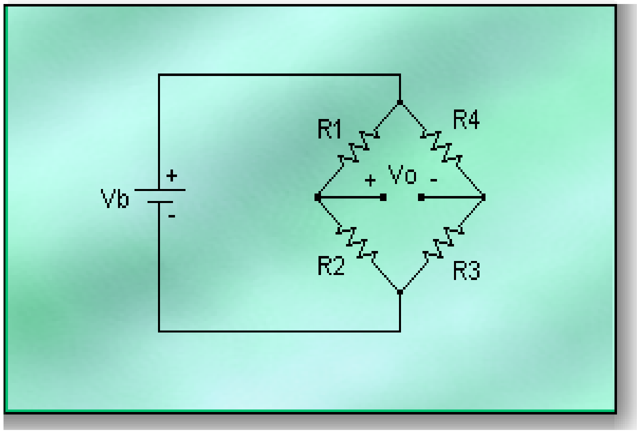


Figure 1-4: The Wheatstone bridge

On the left hand side, *R*1 and *R*2 act as a resistor divider so V_{0+} can be calculated as:

$$V_{0+} = V_b \frac{R2}{R1 + R2} = V_b \frac{1}{1 + (R1)/(R2)}$$

Similarly for the right hand side V_{0-}:

$$V_{0-} = V_b \frac{R3}{R3 + R4} = V_b \frac{1}{1 + (R4)/(R3)}$$

V_0 = *V*+ minus *V*- is therefore:

$$V_0 = V_b \cdot \left(\frac{1}{1 + (R1)/(R2)} - \frac{1}{1 + (R4)/(R3)} \right)$$

If the ratio on one side equals that on the other (*R*2/*R*1 = *R*4/*R*3), the output voltage (V_0) is 0V. The fact that the ratios determine the output enables compensation gages to be used to compensate for bonding and temperature errors.

In the previous figure, if *R*2 was a gage, *R*1 could be used to compensate for some of the errors. However, to do this it should ideally be of the same type and bonded as close as possible to *R*2. It must be bonded perpendicularly to *R*2 so as not to cancel out *R*2 altogether.

The following figure exaggerates how the perpendicular gage experiences a strain of opposite polarity, but of smaller magnitude. This ratio of the transverse magnitudes is known as Poisson's ratio (ν) and for most metals is approximately 0.3.

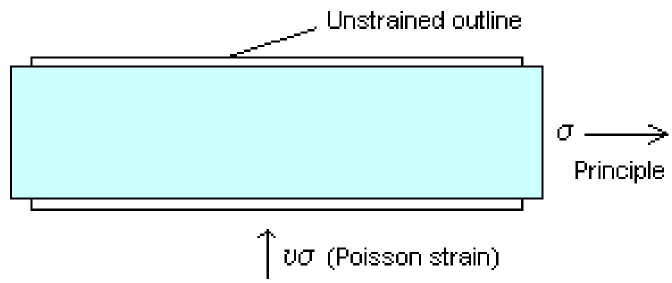


Figure 1-5: An illustration of transverse strain

Sometimes the compensation gage can be mounted so that the strain is the same magnitude but of opposite sign, as with bending beams as displayed in Figure 1-6 on page 6.

1.5 Resistance as a function of fractional change

Previously in this paper, the fractional change in resistance for a resistor was defined as:

$$\delta \equiv \frac{R - R_{nom}}{R_{nom}}$$

This can be rewritten as:

$$R = R_{nom} \cdot (\delta + 1)$$

In the analysis which follows, resistance values of active gages are replaced with the above equation.

In particular, a Poisson gage, when used, is written as:

$$R = R_{nom} \cdot (-\nu\delta + 1)$$

where ν is the Poisson ratio, and the negative sign indicates strain in the opposite direction to the principle axis.

The following figure illustrates the use of Poisson compensation and “opposed” compensation as used on bending beams.

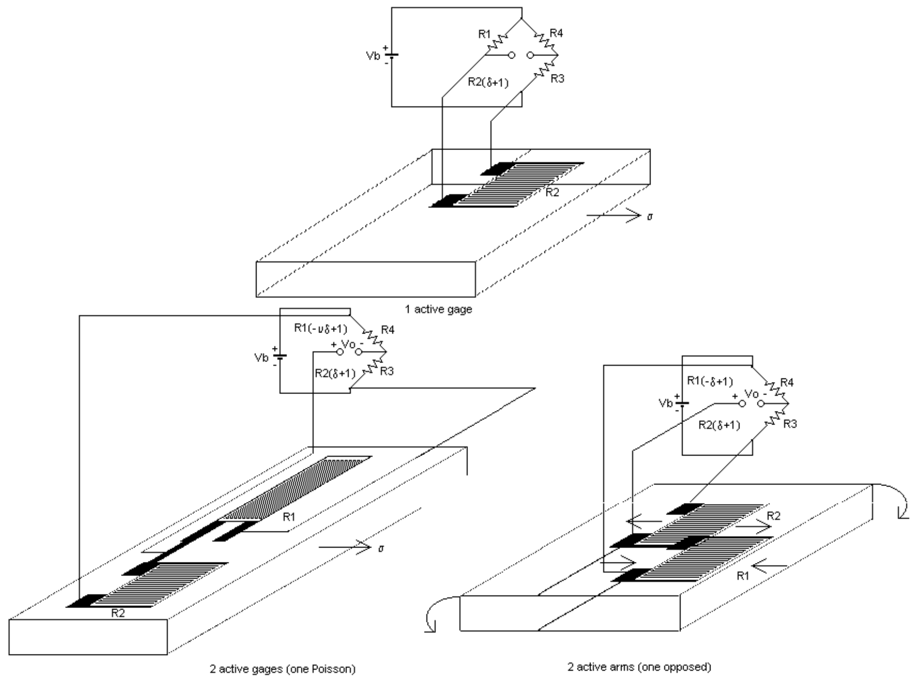


Figure 1-6: An uncompensated gage and two types of compensation

In the next section, the output voltage (V_0) as a function of the fractional change (δ) is examined. The inverse function, δ, is as important a function of V_0.

Knowing the output voltage of a bridge, how is the fractional change causing it, and hence the strain, calculated?

The analysis is carried out using the two active arms with Poisson as shown in the previous figure. One reason for this is the equations for the single gage can be got by setting ν=0 and for the opposed configuration by setting ν=1.

1.6 Equations for two active gages, one of which is Poisson

The following figure displays two active gages, one of which (*R*1) is mounted perpendicular to the uniaxial stress and hence experiences a transverse (Poisson) strain. An amplifier with a gain (*G*) is also displayed.

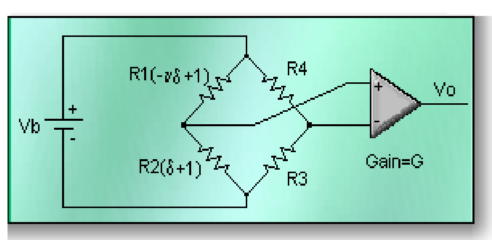


Figure 1-7: Bridge with two active arms (one Poisson) and amplifier

The output voltage of the amplifier is defined as:

$$V_0 = G \cdot V_b \cdot \left(\frac{1}{1 + \frac{R1_{nom}}{R2_{nom}} \cdot \frac{(-\nu\delta + 1)}{(\delta + 1)}} - \frac{1}{1 + \frac{R4}{R3}} \right)$$

Given a bridge that is balanced in the unstrained condition (that is, $R1_{nom}/R2_{nom} = R4/R3 = 1$), the equation becomes:

This is the relationship between V_0 and δ. For small δ this is almost linear and the sensitivity (*S*) can be defined as:

$$V_0 = G \cdot V_b \cdot \left(\frac{(\delta + 1)}{2 + \delta \cdot (1 - \nu)} - \frac{1}{2}\right) = G \cdot V_b \cdot \frac{\delta \cdot (1 + \nu)}{2 \cdot \delta \cdot (1 - \nu) + 4}$$

$$S \equiv Limit \left.\frac{V_0}{\delta}\right|_{\delta \to 0} \Rightarrow S = \frac{G \cdot V_b \cdot (1 + \nu)}{4}$$

From the last two equations, to get the relationship between δ and V_0:

$$\left(V_0 = S \cdot \frac{\delta}{1 + 0.5 \cdot \delta \cdot (1 - \nu)}\right) \Rightarrow \frac{V_0}{S}(1 + 0.5 \cdot \delta \cdot (1 - \nu)) = \delta$$

Bringing the terms with δ to the left gives:

$$\delta \cdot \left(\frac{V_0}{2 \cdot S}(1 - \nu) - 1\right) = -\frac{V_0}{S}$$

Therefore,

$$\delta = \frac{1}{\frac{S}{V_0} - \frac{(1 - \nu)}{2}}$$

A Maclaurin (or Taylor about 0) series expansion gives:

$$\delta = \frac{1}{S}\left(V_0 + 2(1 - \nu) \cdot V_0^2 + 4 \cdot (1 - \nu)^2 \cdot V_0^3 + 8 \cdot (1 - \nu)^3 \cdot V_0^4 + \ldots (2 - 2 \cdot \nu)^{n - 1} \cdot V_0^n\right)$$

These equations have been derived for seven types of bridge as shown in the following table. If the left-hand side arms are swapped with the right-hand side then multiply the sensitivity by -1. If the top arms are swapped with the bottom arms, again multiply the sensitivity by -1.

Table 1-1: Bridge types

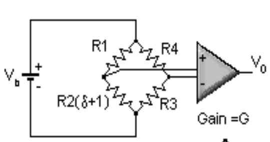
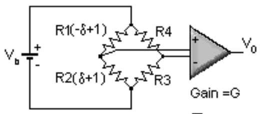
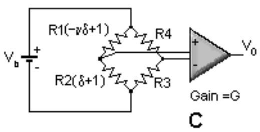
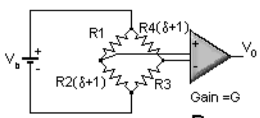
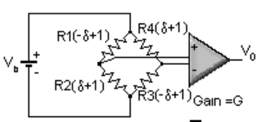
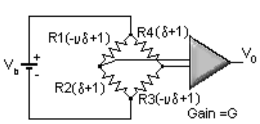
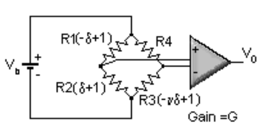
Topology	Description	Sensitivity	Vo(δ)	δ(Vo)
 A	One active gage in uniaxial tension or compression	$S = \frac{G \cdot V_b}{4}$	$V_0 = S\frac{\delta}{1 + 0.5 \cdot \delta}$	$\delta = \left(\frac{S}{V_0} - \frac{1}{2}\right)^{-1}$ $\delta = \frac{1}{S}\sum_{n = 1}^{\infty} 2^{n - 1} \cdot V_0^n$

Table 1-1: Bridge types

Topology	Description	Sensitivity	Vo(δ)	δ(Vo)
 B	Two active gages with equal and opposite strains, typical of bending beam arrangement	$S = \frac{G \cdot V_b}{2}$	$V_0 = S \cdot \delta$	$\delta = \frac{1}{S} V_0$
 C	Two active gages in uniaxial tension or compression, one mounted perpendicular (Poisson=*n*)	$S = \frac{G \cdot V_b \cdot (1 + \nu)}{4}$	$V_0 = S \frac{\delta}{1 + 0.5 \cdot (1 - \nu) \cdot \delta}$	$\delta = \left(\frac{S}{V_0} - \frac{1 - \nu}{2} \right)^{-1}$ $\delta = \frac{1}{S} \sum_{n=1}^{\infty} (2 - 2 \cdot \nu)^{n-1} \cdot V_0^n$
 D	Two active gages, for example, used on opposite sides of column with low temperature gradient	$S = \frac{G \cdot V_b}{2}$	Same as Type A above	Same as Type A above
 E	Four active gages, paired in equal and opposite uniaxial tension or compression	$S = G \cdot V_b$	Same as Type B above	Same as Type B above **(LINEAR)**
 F	Four active gages, in uniaxial tension or compression, two mounted perpendicular (Poisson=*n*)	$S = \frac{G \cdot V_b \cdot (1 + \nu)}{2}$	Same as Type C above	Same as Type C above
 G	Four active gages, Poisson pairs at equal strain but opposite sign, for example, beam	Same as Type F above	Same as Type B above	Same as Type B above **(LINEAR)**

1.7 Straight line approximations

How important are the higher coefficients? The following table lists the approximate errors (in ppm) for assuming first, second, third or fourth order fits, for δ as a function of V_0, for various values of mΩ/Ω:

Table 1-2: Error calculated in respect of expected value

δ=mΩ/Ω	me	1st order	2nd order	3rd order	4th order
100,00	50,000	-50,000	-2750	-150	8
50,000	25,000	-25,000	-600	-20	-0.25
10,000	5,000	-5000	-25	0	0

These errors are calculated with respect to the expected value. In bipolar applications, the range is twice either extreme so the errors with respect to full range is halved:

- With a 10-bit A/D system, one count corresponds to 977ppm (≈ 0.1%)
- With a 12-bit A/D system, one count corresponds to 244ppm (≈ 0.025%)
- With a 16-bit A/D system, one count corresponds to 61ppm

1.8 Conclusion

This paper introduced the basics of strain gage measurement using Wheatstone bridges. Equations were given relating the voltage measured on a bridge to the fractional change in resistance, which in turn is a function of strain.

While some of the sources of errors within the gage itself were briefly discussed, the assumption was made that the excitation and gain circuitry were free from errors.

1.9 References

A series of technical notes on strain gages are available from:

Measurements Group INC.

P.0. Box 2777

Raleigh

North Carolina 27611

USA

Applied Measurement Engineering

Charles P Wright

Prentice Hall

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